MATH2050C Solution to Quiz 1

Justify your answer.

1. (10 marks)

   (a) Use definition to show

   $$\lim_{n\to\infty} \frac{3n^2+5n+2}{2n^2-7} = \frac{3}{2} \ .$$

   Solution We have

   $$\begin{aligned}
   \left| \frac{3n^2+5n+2}{2n^2-7} - \frac{3}{2} \right| &= \frac{10n+25}{2(2n^2-7)} \\
   &= \frac{10n+25}{2(n^2+n^2-7)} \\
   &\le \frac{10n+25}{2n^2} \quad (\text{for } n \ge 3) \\
   &\le \frac{10n+10n}{2n^2} \\
   &\le \frac{10}{n} \ .
   \end{aligned}$$

   For $\varepsilon > 0$, choose $n_0 = \max\{3, [10/\varepsilon]+1\}$. Then for all $n \ge n_0$, $|\frac{3n^2+5n+2}{2n^2-7} - \frac{3}{2}| < \varepsilon$.

   (b) Use Limit Theorem to show the same limit.

   Solution Since $3+5/n+2/n^2 \to 3$ and $2-7/n^2 \to 2$, by the Quotient Rule in the Limit Theorem, we have

   $$\begin{aligned}
   \lim_{n\to\infty} \frac{3n^2+5n+2}{2n^2-7} &= \lim_{n\to\infty} \frac{3+5/n+2/n^2}{2-7/n^2} \\
   &= \frac{\lim_{n\to\infty}(3+5/n+2/n^2)}{\lim_{n\to\infty}(2-7/n^2)} \\
   &= \frac{3}{2} \ .
   \end{aligned}$$

2. (10 marks)

   (a) Show that $\lim_{n\to\infty} a^{1/n} = 1$ where $a > 1$.

   Solution Write $a^{1/n} = 1 + b_n, b_n > 0$. By Bernouli's inequality, $a = (1+b_n)^n \ge 1 + nb_n$ which implies $a - 1 \ge nb_n$ or $0 \le b_n \le (a-1)/n$. By Squeeze Theorem, $0 \le \lim_{n\to\infty} b_n \le \lim_{n\to\infty}(a-1)/n = 0$. Hence $b_n \to 0$ and $a^{1/n} = 1 + b_n \to 1$.

   (b) Find the limit when $0 < a < 1$.

   Solution Write $a = 1/c, c > 1$. By (a) and the Quotient Rule in the Limit Theorem,

   $$\lim_{n\to\infty} a^{1/n} = \lim_{n\to\infty} \frac{1}{c^{1/n}} = \frac{1}{\lim_{n\to\infty} c^{1/n}} = \frac{1}{1} = 1 \ .$$